THE USE OF WALSH FUNCTIONS FOR MULTIPLEXING SIGNALS

by

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Introduction

Walsh functions can be used as carriers of information in a multiplex system such as a system for the simultaneous transmission of a number of telephone signal on a common cable. For this application the orthogonal properties of Walsh functions can be used to maintain the identity of the signals in each channel.

Walsh Functions

Walsh functions are a set of binary functions that are periodic and orthogonal. Examples of the first 15 functions are shown in Figure 1. These functions have been described by Walsh[1], Bell[2] and Harmuth[3] where the set of functions have been sub-divided into two groups called Sal and Cal functions, which correspond to the circular functions Sine and Cosine. However the complete set can be defined by a single function that includes the Sal and Cal functions.

The most compact form of the function is given in Iversons[4] notation as

$$\neq /(A \wedge B) \tag{1}$$

where A and B are integers in binary form, both having the same number of binary bits

$\wedge$ indicates a logical AND function between corresponding bits

$\neq /$ indicates an exclusive OR compression, i.e. adjacent bits are combined by an exclusive OR operation.

This compact expression can be written as

$$(a_n \wedge b_n) \oplus (a_{n-1} \wedge b_{n-1}) \oplus (a_2 \wedge b_2) \oplus (a_1 \wedge b_1) \tag{2}$$

where $a_n \ldots a_1$ and $b_2 \ldots b_1$ are the binary digits of A and B and $\oplus$ represents an exclusive OR function.

This expression gives a logical combination of two binary numbers A and B. If A is an integer k and B is a function of time θ, it is possible to define a Walsh function by the expression

$$\text{WALSH}(k,\theta) = \neq /(k \wedge - \alpha\theta) \tag{3}$$

where α is a factor chosen to make the function periodic in the range $0 < \theta < 1$. The minus sign is chosen before θ to ensure that the functions are identical to the Sal and Cal functions given above.

The above definition can be restated by the following equation, which indicates the logic system that can be used to generate them.

$$\text{WALSH}(k,\theta) = (a_1 s(\theta)) \oplus (a_2 s(2\theta)) \oplus (a_3 s(4\theta)) \oplus \ldots$$
$$\ldots \oplus (a_{n-1} s(2^{n-2}\theta)) \oplus (a_n s(2^{n-1}\theta)) \tag{4}$$

where k is an integer whose binary digits are $a_1, a_2 \cdots a_n$

i.e. $k = 2^{n-1}a_n + 2^{n-2}a_{n-1} + \ldots + 2a_2 + a_1$

and $s(x)$ is a square wave function of x

i.e. $s(x) = 0$ for $0 \le x < \frac{1}{2}$, $1 \le x < 1\frac{1}{2}$, etc.

$\quad\quad = 1$ for $\frac{1}{2} \le x < 1$, $1\frac{1}{2} \le x < 2$, etc.

and $\oplus$ is the exclusive OR function.

i.e. $a \oplus b = 0$ if $a = b$ and $a \oplus b = 1$ if $a \neq b$

The functions defined by this equation are periodic, typified by the range $0 < \theta < 1$. All the functions are synchronized, such that at time $\theta = 0$ all the functions have zero value.

This definition has another important property. As it can be seen from that the first definition it is symmetrical with respect to A and B. The Walsh function is therefore symmetrical between the integer k and the time θ. By interchanging these two variables the same system will give the values of all the orders of Walsh functions at a given time. This property has been described by Shanks[5] and is useful in a system for generating fast Walsh transforms of a function.

Generation of Walsh Functions

Walsh functions can be generated by the system shown by the block diagram in Figure 2. The operation of this can be related to Equation (2): A square wave function on the left is divided by 2 at each stage, enabling the functions $s(\theta)$, $s(2\theta)$ etc. to be generated. These are selected by the switches a_1, a_2 etc. corresponding to the binary bits of the integer k. The signals are combined by the series of exclusive - OR gates. The complete Walsh waveform appears at the right hand side of the system.

Each section of this system can be implemented by the arrangement of logic circuits shown in Figure 3. The JK flip-flop divides

two the incoming square wave. The switch and clusive-OR functions are combined in one group NAND gates. It can be seen that the system n be extended to any order of Walsh function the addition of further identical sections.

The maximum speed of the system is deterined by the propagation time of pulses through e system, and the time difference between the opagation through the chain of Exclusive OR d through the divide by two circuits. Using L integrated circuits as the basic logic elents an eight stage system has been constructed d driven at a frequency of 1 MHz. Under these nditions the output can be changed from WALSH ,θ) up to WALSH (255,θ), the lowest frequency ing 7.8 KHz (WALSH (1,θ) and WALSH (3,θ)) and e highest frequency being 1 MHz (WALSH (128,)). Figure 4 is an oscilloscope trace of a pical waveform as generated by the system.

Multiplexing of Signals

As the Walsh functions are all orthogonal is possible to use them as carriers of formation. If a signal is amplitude modulated a given Walsh function, multiplying it by an entical Walsh function will extract the modation. Multiplying it by any other Walsh nction will result in a zero signal.

The orthogonality of the signals is only tablished if the two Walsh waveforms are nchronized and are in phase. One method of hieving this is to transmit a synchronizing uare wave, the leading edge of this (or the sitive transition) being used to synchronize e Walsh waveform generator at the receiving d. The amplitude of the signal is carried by e trailing edge (or the negative transition) the square wave. In this way the Walsh nction generator can be triggered to its next lue well before the transition containing the gnal is received.

At the end of each cycle the synchronizing gnal is interrupted for approximately 2 μs as own in Figure 5. A reset pulse can be excted from this for initializing the Walsh nerator. This will only be necessary at the ginning of operation, in the event of interption or if synchronization is lost due to ise signals. It can be seen that all the lsh functions are zero immediately following e synchronizing pulse, and therefore no sig- l information is carried at that time. This ll enable all the waveforms to be correctly ased by the time the first information signal rives.

The modulation process consists of genering one cycle similar to the synchronizing veform, but having an amplitude equal to that the signal whenever the Walsh function is ne', and having zero amplitude if the Walsh nction is 'zero'. Examples of modulated gnals are shown in Figure 6. In the demodution process the amplitude of the negative nsition of the waveform is extracted and her added or subtracted from the output pending on whether the Walsh function is 'one' 'zero'.

System Performance

Study of the model of the system gives the performance to be expected, and two possible sources of cross modulation become evident. The first arises from unbalance in the receiving circuit, and the second from loss of phase synchronization.

The rejection of signals carried by other channel Walsh functions depends on the cancellation of the signal after it has been 'multiplied' by a different Walsh function. This involves the adding together of a number of signals, with suitable sign changes, to give a zero result. This requires careful circuit design for satisfactory signal performance to be achieved under working conditions.

A small shift of phase synchronization would superimpose the information in a given time slot on the next time slot. Poor repeater response or lack of phase equalization would give a similar result. The effect of this has been simulated by a computer program. It was shown that the magnitude of the cross modulation was small except in the case of certain pairs of channels. These channel pairs have the same order of Walsh functions except for the first binary bit, i.e. channels 251 and 123. However, if these pairs of channels are chosen in the same way as cable pairs in a multi-wire cable, and balanced signals placed on them, the effect of cross modulation becomes less important. This has the added advantage that signals do not amplitude modulate the carrier, thereby reducing the cross modulation due to non-linear components in the system.

Conclusion

Parts of an experimental setup have been built in order to test the critical parts of it. Furthermore study of the model should give its expected characteristics, including such secondary parameters as the effects of phase delay and linearity of repeater amplifiers, signal clipping, channel noise, etc. However, the final check must come from fully engineered systems with its performance evaluated under operating conditions.

References

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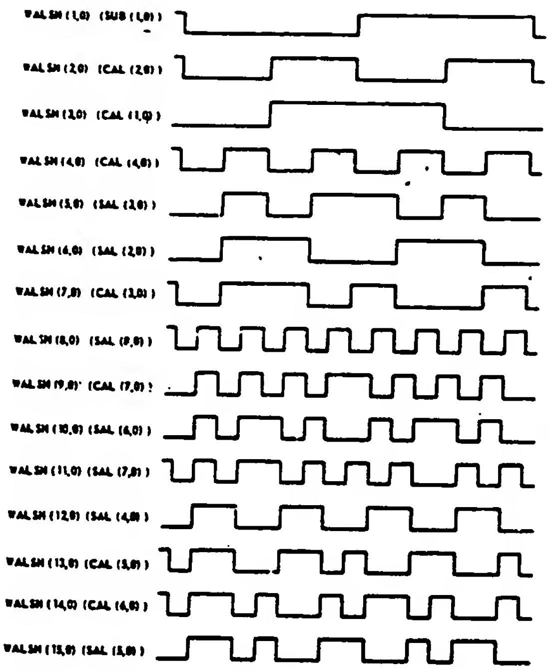

FIGURE 1

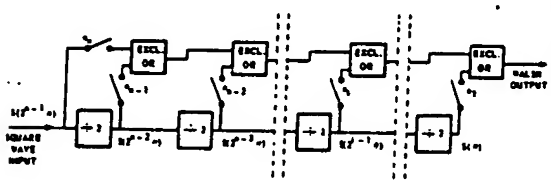

FIGURE 2

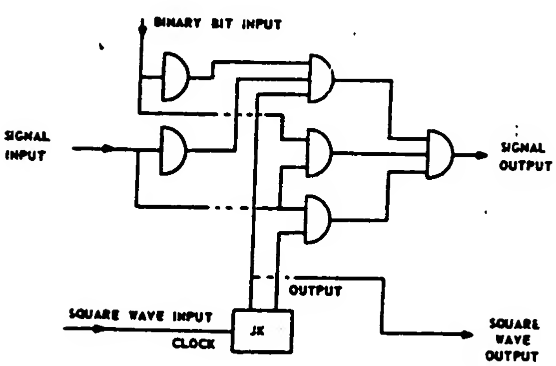

FIGURE 3

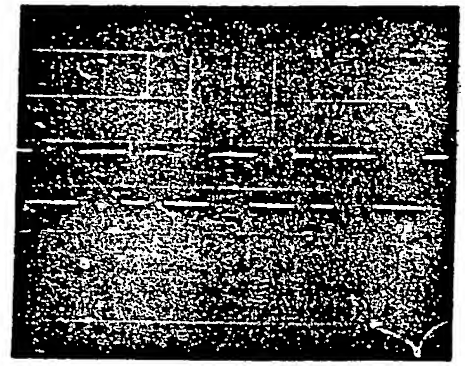

Figure 4 Walsh Waveform

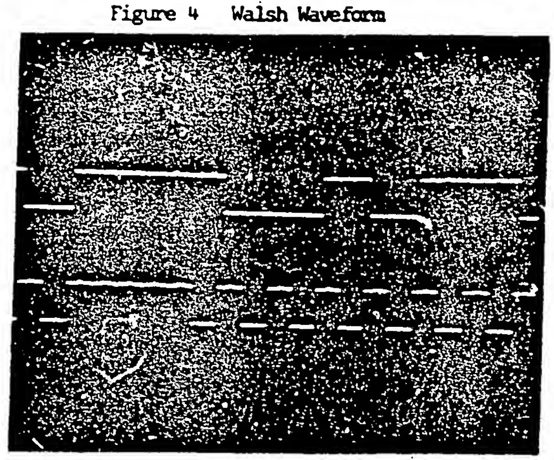

Figure 5 a) Walsh Waveform
b) Synchronizing Waveform

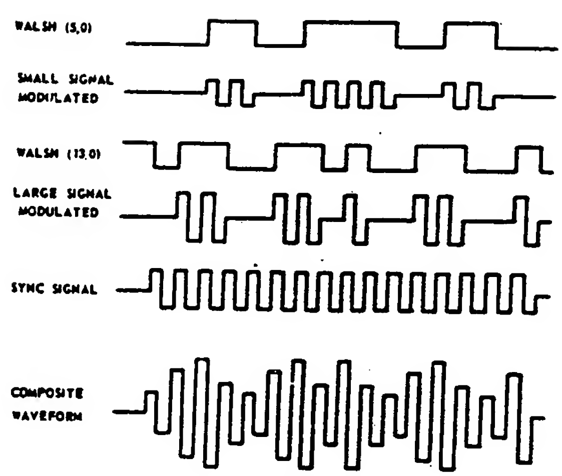

FIGURE 6